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Astrophysical and cosmological tests of quantum theory

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Abstract
We discuss several proposals for astrophysical and cosmological tests of quantum theory. The tests are motivated by deterministic hidden-variables theories, and in particular by the view that quantum physics is merely an effective theory of an equilibrium state. The proposed tests involve searching for nonequilibrium violations of quantum theory in: primordial inflaton fluctuations imprinted on the cosmic microwave background, relic cosmological particles, Hawking radiation, photons with entangled partners inside black holes, neutrino oscillations and particles from very distant sources.

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Dedicated to Professor G-C Ghirardi on the occasion of his seventieth birthday.

1. Introduction

It is important that we continue to test quantum theory in new and extreme conditions: as with any scientific theory, its domain of validity can be determined only by experiment. For this purpose, it is helpful to have theories that agree with quantum theory in some limit, and deviate from it outside that limit. Examples of such theories include models of wavefunction collapse, pioneered by Pearle [1–3] and by Ghirardi, Rimini and Weber [4], and hidden-variables theories with nonstandard probability distributions ('quantum nonequilibrium'), advocated in particular by the author [5–13].

While it is possible that quantum theory might turn out to break down in a completely unexpected way, and in a completely unexpected place, the chances of a successful detection of a breakdown would seem higher, the better motivated the theory describing the breakdown.

For some 25 years, extreme tests of quantum theory focused mostly on experiments demonstrating violations of Bell's inequality. These tests were well motivated: at the time (say

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in the 1970s), it was reasonable to suspect that locality might force a deviation from quantum correlations for entangled states of widely separated systems. However, as the evidence for violations of Bell's inequalities accumulated, the long-range correlations predicted by quantum theory came to be widely accepted as a fact of nature, and the known domain of validity of quantum theory was extended into an important region.

Tests of collapse models again stem from a compelling motivation: to test the superposition of quantum states as far as possible into the macroscopic regime. Will a sufficiently macroscopic superposition decay via corrections to the Schrödinger equation? Such experiments are still being carried out, and again (for as long as they prove negative) extend our confidence in the validity of quantum theory in an important way.

In this paper, we discuss a number of new proposals for extreme tests of quantum theory, proposals that are motivated by thinking about quantum physics from the point of view of deterministic hidden variables.

A deterministic hidden-variables theory provides a mapping $\omega = \omega(M, \lambda)$ from initial ('hidden') parameters λ to outcomes ω of a quantum experiment (or 'measurement') specified by the settings M of macroscopic equipment. In addition, in order to make contact with the statistics observed over an ensemble of similar experiments (with fixed M and variable λ), it must be assumed that over an ensemble the hidden variables λ have some distribution $\rho(\lambda)$, so that (for example) the expectation value of ω will be given by

$$\langle\omega\rangle = \int \mathrm{d}\lambda\, \rho(\lambda)\omega(M, \lambda). \tag{1}$$

For the hidden-variables theory to provide a successful account of quantum phenomena, there must exist a particular distribution $\rho_{\mathrm{QT}}(\lambda)$ such that all corresponding expectation values $\langle\omega\rangle_{\mathrm{QT}}$ match the prediction $\langle\omega\rangle_{\mathrm{QT}} = \mathrm{Tr}(\hat{\rho}\hat{\Omega})$ of standard quantum theory (for some density operator $\hat{\rho}$ and 'observable' $\hat{\Omega}$).

A concrete example is provided by the pilot-wave theory of de Broglie [14] and Bohm [15].[2] There, the outcome of a single run of an experiment is determined by the initial ('hidden') configuration $X(0)$ of the system, together with the initial guiding wavefunction $\Psi(X, 0)$, so that λ consists of the pair $X(0)$, $\Psi(X, 0)$. For an ensemble with the same $\Psi(X, 0)$ (and the same apparatus settings M), we have $\lambda = X(0)$, and the quantum equilibrium distribution $\rho_{\mathrm{QT}}(\lambda)$ is given by $P_{\mathrm{QT}}(X, 0) = |\Psi(X, 0)|^2$.

It is not usually appreciated that the distribution $\rho_{\mathrm{QT}}(\lambda)$ is conceptually quite distinct from the mapping $\omega = \omega(M, \lambda)$. The latter is a property of each individual run of the experiment, specifying the 'dynamics' whereby each value of λ determines an outcome ω; while the former is a property of the ensemble, specifying the distribution of 'initial conditions' for the parameters λ. As we have argued at length elsewhere [5–13], if one takes deterministic hidden-variables theories seriously, one must conclude that quantum theory is merely the phenomenology of a special 'quantum equilibrium' distribution $\rho_{\mathrm{QT}}(\lambda)$. In principle, there exists a wider physics beyond the domain of quantum theory, with 'nonequilibrium' distributions $\rho(\lambda) \neq \rho_{\mathrm{QT}}(\lambda)$ and non-quantum expectation values $\langle\omega\rangle \neq \langle\omega\rangle_{\mathrm{QT}}$. This paper concerns the possibility of detecting such deviations from quantum theory, through astrophysical and cosmological observations.

[2] At the 1927 Solvay conference, de Broglie proposed what we now know as the first-order pilot-wave dynamics of a (nonrelativistic) many-body system, with a guiding wave in configuration space determining the particle velocities, and he applied it to simple quantum phenomena such as interference, diffraction and atomic transitions. In 1952, Bohm showed that the general quantum theory of measurement was a consequence of de Broglie's dynamics (when applied to an initial equilibrium ensemble), even though Bohm actually wrote the dynamics in a pseudo-Newtonian or second-order form based on acceleration. For a detailed analysis of de Broglie's construction of pilot-wave theory, as well as for a full discussion of the respective contributions of de Broglie and Bohm, see [16] (which also includes an English translation of de Broglie's 1927 Solvay report).

2. Quantum nonequilibrium: what, when and where?

What exactly should one look for? Quantum nonequilibrium opens up an immense range of possible new phenomena. Here, we focus on deviations from the following quintessentially quantum effects:

- Single-particle interference. For example, in a double-slit experiment with particles of wavelength $2\pi/k$, incident on a screen with slits separated by a distance a, at large distances behind the screen quantum theory predicts a modulation

$$|\psi(\theta)|^2 \propto \cos^2\left(\tfrac{1}{2}ka\theta\right) \tag{2}$$

  in the distribution of single-particle detections at angular deviation θ (measured from the normal to the screen). If the experiment is performed with one particle at a time, each outcome θ will (in a hidden-variables theory) be determined by a mapping $\theta = \theta(M, \lambda)$ (where again M specifies the experimental arrangement). The quantum distribution (2) will correspond to the quantum equilibrium distribution $\rho_{\mathrm{QT}}(\lambda)$, while nonequilibrium $\rho(\lambda) \neq \rho_{\mathrm{QT}}(\lambda)$ will generally imply deviations from (2)—for example, an anomalous blurring of the interference fringes.
- Malus' law for two-state systems. For example, for single photons incident on a polarizer, quantum theory predicts a modulation

$$p^{+}_{\mathrm{QT}}(\Theta) = \tfrac{1}{2}(1 + P\cos 2\Theta) \tag{3}$$

  of the transmission probability, where P is the (ensemble) polarization of the beam and Θ is the angle of the polarizer. (For $P = 1$, $p^{+}_{\mathrm{QT}}(\Theta) = \cos^2\Theta$.) As shown elsewhere [12], Malus' law (3) is equivalent to the additivity of expectation values for non-commuting observables in a two-state system, and such additivity generically breaks down in quantum nonequilibrium. Deviations from (3) then provide a convenient signature of nonequilibrium.
- Gaussian vacuum fluctuations. Standard quantum field theory predicts that seemingly empty space is the seat of field fluctuations corresponding to a Gaussian random process, with a specified variance for each mode $\mathbf{k}$. Quantum nonequilibrium for vacuum fields will generically imply a departure from Gaussianity and deviations from the predicted variance (or width) for each $\mathbf{k}$.

The possible breakdown of Malus' law deserves special comment. Any two-state quantum system has observables $\hat{\sigma} \equiv \mathbf{m}\cdot\hat{\boldsymbol{\sigma}}$ taking values $\sigma = \pm 1$, where $\mathbf{m}$ is a unit vector in Bloch space and $\hat{\boldsymbol{\sigma}}$ is a Pauli spin operator. Quantum theory predicts that, for an ensemble with density operator $\hat{\rho}$, the probability $p^{+}_{\mathrm{QT}}(\mathbf{m})$ for an outcome $\sigma = +1$ of a quantum measurement of $\hat{\sigma}$ is given by

$$p^{+}_{\mathrm{QT}}(\mathbf{m}) = \tfrac{1}{2}(1 + \langle\hat{\sigma}\rangle) = \tfrac{1}{2}(1 + \mathbf{m}\cdot\mathbf{P}), \tag{4}$$

where $\mathbf{P} = \langle\hat{\boldsymbol{\sigma}}\rangle = \mathrm{Tr}(\hat{\rho}\hat{\boldsymbol{\sigma}})$ is the mean polarization. (For photons, an angle θ on the Bloch sphere corresponds to a physical angle $\Theta = \theta/2$.) It is specifically the linearity in $\mathbf{m}$ of the quantum expectation value

$$E_{\mathrm{QT}}(\mathbf{m}) \equiv \langle\mathbf{m}\cdot\hat{\boldsymbol{\sigma}}\rangle = \mathrm{Tr}(\hat{\rho}\,\mathbf{m}\cdot\hat{\boldsymbol{\sigma}}) = \mathbf{m}\cdot\mathbf{P}$$

that is equivalent to expectation additivity for incompatible observables. The proof is straightforward [12]. For an arbitrary unit vector $\mathbf{m} = \sum_i c_i\mathbf{m}_i$, where $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3\}$ is an orthonormal basis in Bloch space, expectation additivity implies that $E_{\mathrm{QT}}(\mathbf{m}) = \sum_i c_i E_{\mathrm{QT}}(\mathbf{m}_i)$. Invariance of $E_{\mathrm{QT}}(\mathbf{m})$ under a change of basis $\mathbf{m}_i \rightarrow \mathbf{m}'_i$ then implies that $E_{\mathrm{QT}}(\mathbf{m}) = \mathbf{m}\cdot\mathbf{P}$ where $\mathbf{P} \equiv \sum_i E_{\mathrm{QT}}(\mathbf{m}_i)\mathbf{m}_i$ is a vector with norm $0 \leqslant P \leqslant 1$. Using expectation additivity again, we have $\mathbf{P} = \langle\hat{\boldsymbol{\sigma}}\rangle$.

A deterministic hidden-variables theory applied to a two-state system will provide a mapping $\sigma = \sigma(\mathbf{m}, \lambda)$ that determines the measurement outcomes $\sigma = \pm 1$. As shown in [12], for an arbitrary distribution $\rho(\lambda) \neq \rho_{\mathrm{QT}}(\lambda)$ of hidden variables λ the nonequilibrium expectation value

$$E(\mathbf{m}) \equiv \langle\sigma(\mathbf{m}, \lambda)\rangle = \int \mathrm{d}\lambda\, \rho(\lambda)\sigma(\mathbf{m}, \lambda)$$

will generally *not* take the linear form $\mathbf{m}\cdot\mathbf{P}$ for some Bloch vector $\mathbf{P}$, and the nonequilibrium outcome probability

$$p^{+}(\mathbf{m}) = \tfrac{1}{2}(1 + E(\mathbf{m})) \tag{5}$$

will generally not take the quantum form (4). Both the linearity and the additivity are generically violated in quantum nonequilibrium.

A natural parameterization of nonequilibrium outcome probabilities $p^{+}(\mathbf{m})$ for two-state systems may be obtained by expanding $p^{+}(\mathbf{m})$ in terms of spherical harmonics, with the unit vector $\mathbf{m}$ specified by angular coordinates (θ, ϕ) on the Bloch sphere. For example, a probability law that includes a quadrupole term,

$$p^{+}(\mathbf{m}) = \tfrac{1}{2}(1 + \mathbf{m}\cdot\mathbf{P} + (\mathbf{m}\cdot\mathbf{b})(\mathbf{m}\cdot\mathbf{P})) \tag{6}$$

(for some non-zero vector $\mathbf{b}$), corresponding to a *nonlinear* expectation value

$$E(\mathbf{m}) = \mathbf{m}\cdot\mathbf{P} + (\mathbf{m}\cdot\mathbf{b})(\mathbf{m}\cdot\mathbf{P}), \tag{7}$$

would signal a failure of expectation additivity and a violation of quantum theory.

More generally, one might consider nonlinear expectation values

$$E(\mathbf{m}) = m_i P_i + m_i m_j Q_{ij} + m_i m_j m_k R_{ijk} + \cdots$$

(summing over repeated indices), where $Q_{ij}, R_{ijk}, \ldots$ are tensors in Bloch space. The experimental challenge is to set upper bounds on the magnitudes $|Q_{ij}|, |R_{ijk}|, \ldots$, for systems in extreme conditions. The theoretical challenge, of course, is to provide precise predictions for $Q_{ij}, R_{ijk}, \ldots$.

The statistical predictions of quantum theory and of quantum field theory have of course been verified in countless experiments. For two-state systems, for example, all known experimental data are consistent with $Q_{ij} = R_{ijk} = \cdots = 0$. From a hidden-variables perspective, however, there are good reasons to *expect* that the experiments performed so far yield agreement with quantum theory. This is because all the experiments performed so far have been done with systems that have had a long and violent astrophysical history. Atoms in the laboratory, for example, have a history stretching back to the formation of stars, or even earlier (to big bang nucleosynthesis), during which these atoms have undergone numerous complex interactions with other systems. Every degree of freedom we have access to has a complex past history of interaction with other degrees of freedom, a history that ultimately merges with the history of the early universe. This fact is highly significant, because it suggests that the quantum equilibrium distribution $\rho_{\mathrm{QT}}(\lambda)$ observed today could have emerged from past interactions, via a process of relaxation (analogous to relaxation to thermal equilibrium in ordinary physics).

Relaxation to quantum equilibrium has been studied in some detail for the case of pilot-wave theory. The quantity $H = \int \mathrm{d}X\, P \ln(P/|\Psi|^2)$ (equal to minus the relative entropy of an arbitrary distribution P with respect to $|\Psi|^2$) obeys a coarse-graining H-theorem analogous to the classical one [5, 7, 9]; and numerical simulations for simple two-dimensional systems

[17, 18] show a rapid (approximately exponential) decay of the coarse-grained H-function, $\bar{H}(t) \to 0$, with a corresponding coarse-grained relaxation $\bar{P} \to \overline{|\Psi|^2}$ (given appropriate initial conditions on P and Ψ, see [9]).

In pilot-wave theory, then, given the known past history of the universe, there is every reason to expect the systems being examined today to be in quantum equilibrium. Presumably, similar conclusions would hold in any reasonable (deterministic) hidden-variables theory: we expect that the known past interactions will generate a similar relaxation $\rho(\lambda) \to \rho_{\mathrm{QT}}(\lambda)$.

In this scenario, quantum theory is merely an effective theory, describing the physics of an equilibrium state that emerged some time in the remote past. Considering this scenario further suggests clues as to where quantum theory might break down.

The obvious place to look is the very early universe. At sufficiently early times, quantum nonequilibrium $\rho(\lambda) \neq \rho_{\mathrm{QT}}(\lambda)$ may have still existed. How can one probe such early times experimentally? One possibility is provided by inflationary cosmology, according to which primordial vacuum fluctuations in a scalar field ϕ (present during an early period of exponential spatial expansion) are responsible for the early inhomogeneities that seeded the formation of large-scale structure in the universe and that left an observable imprint on the cosmic microwave background (CMB). This suggests that primordial quantum nonequilibrium could have a measurable effect on the CMB temperature anisotropy [19]. Another possibility is based on the idea [9] that certain particle species may have decoupled so early that they did not have time to reach quantum equilibrium: such nonequilibrium relic particles could still exist today. One is then led to consider testing quantum theory for relic particles from very early times.

Instead of looking for residual nonequilibrium from the distant past, would it be possible to *generate* nonequilibrium today? It has been suggested [20] that gravitation may be capable of generating quantum nonequilibrium. In particular, information loss in black holes might be avoided if Hawking radiation consisted of nonequilibrium particles, since the final state could then contain more information than the conventional (quantum) final state. Following this line of reasoning, one is led to suggest that if one half of a bipartite entangled state fell behind the event horizon of a black hole, the other half would evolve away from quantum equilibrium. Such a situation might occur naturally via atomic cascade emissions in black-hole accretion discs.

There are also theoretical reasons for suspecting that quantum-gravitational effects could induce pure-to-mixed transitions in, for example, oscillating neutrinos. Motivated once again by the possible avoidance of information loss, such transitions might be accompanied by the generation of quantum nonequilibrium.

Finally, the possibility of gravitational effects generating nonequilibrium at the Planck scale motivates us to consider tests of quantum probabilities at very small lengthscales, for all particles whatever their origin. As we shall see, in the right circumstances the spreading of wave packets for particles emitted by remote sources can act as a cosmological 'microscope', expanding tiny deviations from quantum theory to observable scales.

We shall now examine these suggestions in turn.

3. Inflation as a test of quantum theory in the early universe

The temperature anisotropy $\Delta T(\theta, \phi) \equiv T(\theta, \phi) - \bar{T}$ of the microwave sky may be expanded in terms of spherical harmonics as

$$\frac{\Delta T(\theta, \phi)}{\bar{T}} = \sum_{l=2}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta, \phi). \tag{8}$$

It is usual to regard the observed $T(\theta, \phi)$ as a realization of a stochastic process, such that the underlying probability distribution for each coefficient a_{lm} is independent of m (as follows if the probability distribution for $T(\theta, \phi)$ is assumed to be rotationally invariant). For large enough l, the (theoretical) ensemble average $\langle |a_{lm}|^2 \rangle$ may then be accurately estimated as

$$\langle |a_{lm}|^2 \rangle \approx \frac{1}{2l+1} \sum_{m=-l}^{+l} |a_{lm}|^2 \equiv C_l. \tag{9}$$

The anisotropy $\Delta T(\theta, \phi)$ is believed to have been produced by (classical) inhomogeneities on the last scattering surface (when CMB photons decoupled from matter). There is a well-established theory expressing the a_{lm} in terms of a Fourier-transformed 'primordial curvature perturbation' $\mathcal{R}_{\mathbf{k}}$ (see, for example [19, 21] for details). Assuming that the underlying probability distribution for $\mathcal{R}_{\mathbf{k}}$ is translationally invariant, it may be shown that

$$C_l = \frac{1}{2\pi^2} \int_0^\infty \frac{\mathrm{d}k}{k} \mathcal{T}^2(k, l) \mathcal{P}_{\mathcal{R}}(k), \tag{10}$$

where $\mathcal{T}$ is a function encoding the relevant astrophysical processes and

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{V} \langle |\mathcal{R}_{\mathbf{k}}|^2 \rangle \tag{11}$$

is the primordial power spectrum for $\mathcal{R}_{\mathbf{k}}$ (with V a normalization volume). Data for the C_l suggest that $\mathcal{P}_{\mathcal{R}}(k) \approx$ const (an approximately scale-free spectrum) [22].

Now, inflationary cosmology predicts that $\mathcal{R}_{\mathbf{k}}$ is given by [21]

$$\mathcal{R}_{\mathbf{k}} = -\left[\frac{H}{\dot{\phi}_0} \phi_{\mathbf{k}} \right]_{t=t_*(k)}, \tag{12}$$

where $H \equiv \dot{a}/a$ is the (approximately constant) Hubble parameter of the inflating universe (with metric $\mathrm{d}\tau^2 = \mathrm{d}t^2 - a^2\,\mathrm{d}\mathbf{x}^2$ and scale factor $a = a(t)$), ϕ_0 and ϕ are respectively the spatially homogeneous and inhomogeneous parts of the inflaton field, and the right-hand side is evaluated at a time $t_*(k)$ a few e-folds after the (exponentially expanding) physical wavelength $\lambda_{\mathrm{phys}} = 2\pi a(t)/k$ of the mode $\mathbf{k}$ exceeds (or 'exits') the Hubble radius H^{-1}. To a first approximation, inflation predicts that $\phi_{\mathbf{k}}$ will have (at time $t_*(k)$) a quantum variance

$$\langle |\phi_{\mathbf{k}}|^2 \rangle_{\mathrm{QT}} = \frac{V}{2(2\pi)^3} \frac{H^2}{k^3} \tag{13}$$

and a scale-invariant power spectrum

$$\mathcal{P}_{\phi}^{\mathrm{QT}}(k) \equiv \frac{4\pi k^3}{V} \langle |\phi_{\mathbf{k}}|^2 \rangle_{\mathrm{QT}} = \frac{H^2}{4\pi^2} \tag{14}$$

(where $\langle |\phi_{\mathbf{k}}|^2 \rangle_{\mathrm{QT}}$ is obtained from the Bunch–Davies vacuum in de Sitter space, for $\lambda_{\mathrm{phys}} \gg H^{-1}$). This results in a scale-free spectrum (in the slow-roll limit $\dot{H} \to 0$) for $\mathcal{R}_{\mathbf{k}}$,

$$\mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k) = \frac{1}{4\pi^2} \left[\frac{H^4}{\dot{\phi}_0^2} \right]_{t_*(k)}, \tag{15}$$

in approximate agreement with what is observed.

Quantum nonequilibrium in the Bunch–Davies vacuum would yield deviations from (13). Further, in the pilot-wave version of quantum field theory, it may be shown [19] that any (microscopic) quantum nonequilibrium that is present at the onset of inflation will be *preserved* during the inflationary phase (instead of relaxing), and will in fact be transferred to macroscopic lengthscales by the growth of physical wavelengths $\lambda_{\mathrm{phys}} \propto a(t) \propto e^{Ht}$.

This is shown by calculating the de Broglie–Bohm trajectories for the inflaton field. Writing $\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}(q_{\mathbf{k}1} + \mathrm{i}q_{\mathbf{k}2})$ (for real $q_{\mathbf{k}r}$, $r = 1, 2$), the Bunch–Davies wavefunctional takes the product form $\Psi[q_{\mathbf{k}r}, t] = \prod_{\mathbf{k}r} \psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t)$, and the de Broglie equation of motion for $q_{\mathbf{k}r}$ is

$$\frac{\mathrm{d}q_{\mathbf{k}r}}{\mathrm{d}t} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}r}}{\partial q_{\mathbf{k}r}},$$

where $\psi_{\mathbf{k}r} = |\psi_{\mathbf{k}r}|\,\mathrm{e}^{\mathrm{i}s_{\mathbf{k}r}}$. Using the known form for $\psi_{\mathbf{k}r}$, it is found that

$$\frac{\mathrm{d}q_{\mathbf{k}r}}{\mathrm{d}t} = -\frac{k^2 H q_{\mathbf{k}r}}{k^2 + H^2 a^2},$$

which has the solution

$$q_{\mathbf{k}r}(\eta) = q_{\mathbf{k}r}(0)\sqrt{1 + k^2\eta^2},$$

where $\eta = -1/Ha$ is the conformal time (running from $-\infty$ to 0). Given this solution for the trajectories, one may easily construct the exact evolution of an arbitrary distribution $\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)$ (generally $\neq |\psi_{\mathbf{k}r}(q_{\mathbf{k}r}, \eta)|^2$). The time evolution amounts to a homogeneous contraction of both $|\psi_{\mathbf{k}r}|^2$ and $\rho_{\mathbf{k}r}$. At times $\eta < 0$, $|\psi_{\mathbf{k}r}|^2$ is a contracting Gaussian packet of width $\Delta_{\mathbf{k}r}(\eta) = \Delta_{\mathbf{k}r}(0)\sqrt{1 + k^2\eta^2}$. In the late-time limit $\eta \to 0$, $|\psi_{\mathbf{k}r}|^2$ approaches a static Gaussian of width $\Delta_{\mathbf{k}r}(0) = H/\sqrt{2k^3}$. At times $\eta < 0$, $\rho_{\mathbf{k}r}$ is a contracting arbitrary distribution of width $D_{\mathbf{k}r}(\eta) = D_{\mathbf{k}r}(0)\sqrt{1 + k^2\eta^2}$ (with arbitrary $D_{\mathbf{k}r}(0)$). In the late-time limit $\eta \to 0$, $\rho_{\mathbf{k}r}$ approaches a static packet of width $D_{\mathbf{k}r}(0)$ (the asymptotic packet differing from the earlier packet by a homogeneous rescaling of q). We then have the result

$$\frac{D_{\mathbf{k}r}(t)}{\Delta_{\mathbf{k}r}(t)} = (\text{const in time}) \equiv \sqrt{\xi(k)}, \tag{16}$$

where for simplicity we assume that (like $\Delta_{\mathbf{k}r}$) the nonequilibrium width $D_{\mathbf{k}r}$ depends on k and t only. (For each mode, the factor $\xi(k)$ may be defined at any convenient fiducial time.) Thus, for each mode $\mathbf{k}$, the widths of the nonequilibrium and equilibrium distributions remain in a fixed ratio over time.

Thus, at least to a first approximation (treating the inflationary phase as an exact de Sitter expansion), if quantum nonequilibrium exists at early times it will not relax during the inflationary phase. Instead, it will indeed be preserved, and be transferred to macroscopic scales by the expansion of physical wavelengths λ_{phys}. This process is especially striking in the late-time limit, where both $\rho_{\mathbf{k}r}$ and $|\psi_{\mathbf{k}r}|^2$ become static. Once the mode exits the Hubble radius, the nonequilibrium becomes 'frozen', while λ_{phys} continues to grow exponentially. The 'frozen' nonequilibrium then corresponds to a physical lengthscale that grows exponentially with time, from microscopic to macroscopic scales. And of course, once inflation has ended, curvature perturbations $\mathcal{R}_{\mathbf{k}}$ at macroscopic lengthscales are transferred to cosmological lengthscales by the subsequent (post-inflationary) Friedmann expansion.

Writing the nonequilibrium variance as

$$\langle|\phi_{\mathbf{k}}|^2\rangle = \langle|\phi_{\mathbf{k}}|^2\rangle_{\text{QT}}\,\xi(k), \tag{17}$$

the resulting power spectrum for $\mathcal{R}_{\mathbf{k}}$ is then just the usual result (15) multiplied by the factor $\xi(k)$:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{\xi(k)}{4\pi^2}\left[\frac{H^4}{\dot{\phi}_0^2}\right]_{t_*(k)}. \tag{18}$$

Early quantum nonequilibrium will generally break the scale invariance of the primordial power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ (at least in pilot-wave theory). Measurements of the angular power spectrum C_l of the microwave sky may be used—in the context of inflationary theory—to constrain the primordial 'nonequilibrium function' $\xi(k)$ [19].

Other measurable effects of early nonequilibrium include violation of the scalar–tensor consistency relation, non-Gaussianity and non-random primordial phases [19].

Work in progress attempts to predict features of the function $\xi(k)$, by studying the evolution of nonequilibrium in an assumed pre-inflationary era: preliminary results suggest that, at the beginning of inflation, nonequilibrium is more likely to have survived at large wavelengths (small k).

Note that primordial nonequilibrium $\xi(k) \neq 1$ might be generated during the inflationary phase by novel gravitational effects at the Planck scale (see section 5)—as well as, or instead of, being a remnant of an earlier nonequilibrium epoch.

Finally, we remark that Perez *et al* [23] have also considered modifying quantum theory in an inflationary context. Their primary motivation is the quantum measurement problem (which is of course especially severe in cosmology). In particular, they discuss how predictions for the CMB could be affected by a dynamical collapse of the wavefunction in the early universe.

4. Relic nonequilibrium particles

The early universe contains a mixture of effectively massless (relativistic) particles. According to the standard analysis, relaxation to thermal equilibrium between different particle species depends on two competing effects: interactions driving different species towards mutual equilibrium, and spatial expansion making different species fall out of mutual equilibrium. Relaxation occurs only if the former overcomes the latter, that is, only if the mean free time t_{col} between collisions is smaller than the timescale $t_{\mathrm{exp}} \equiv a/\dot{a}$ of spatial expansion. In a Friedmann model (perhaps pre- or post-inflationary), $a \propto t^{1/2}$ and $t_{\mathrm{exp}} \propto 1/T^2$ (where T is the photon temperature). Thus, if $t_{\mathrm{col}} = t_{\mathrm{col}}(T)$ falls off slower than $1/T^2$, at sufficiently high temperatures $t_{\mathrm{col}} \gtrsim t_{\mathrm{exp}}$ and thermal equilibrium between the species will not be achieved—or at least, not until the temperature has dropped sufficiently for $t_{\mathrm{col}} \lesssim t_{\mathrm{exp}}$ to hold. Similarly, species that are in thermal equilibrium will subsequently decouple if t_{col} becomes larger than t_{exp} as the universe expands and T decreases (as occurs for CMB photons at recombination). The thermal history of the universe then depends crucially on the functions $t_{\mathrm{col}}(T)$, which in turn depend on the relevant scattering cross sections.

We expect that relaxation to quantum equilibrium in an expanding universe will likewise depend on two competing effects: the usual relaxation seen in flat spacetime, and the stretching of the nonequilibrium lengthscale caused by spatial expansion [9].

As already mentioned, numerical simulations in pilot-wave theory show a very efficient relaxation for systems with two degrees of freedom (given appropriate initial conditions). These simulations were carried out on a static background (flat) spacetime, with a wavefunction equal to a superposition of many different energy eigenstates, for nonrelativistic particles in a two-dimensional box [17] and in a two-dimensional harmonic oscillator potential [18]. The latter case is mathematically equivalent to that of a single decoupled mode $\mathbf{k}$ of a free scalar field on Minkowski spacetime: again writing $\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}(q_{\mathbf{k}1} + \mathrm{i}q_{\mathbf{k}2})$ as above, the wavefunction $\psi_{\mathbf{k}} = \psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)$ of the mode satisfies

$$\mathrm{i}\frac{\partial \psi_{\mathbf{k}}}{\partial t} = -\frac{1}{2}\left(\frac{\partial^2}{\partial q_{\mathbf{k}1}^2} + \frac{\partial^2}{\partial q_{\mathbf{k}2}^2}\right)\psi_{\mathbf{k}} + \frac{1}{2}k^2\left(q_{\mathbf{k}1}^2 + q_{\mathbf{k}2}^2\right)\psi_{\mathbf{k}}, \tag{19}$$

and the de Broglie velocities for $q_{\mathbf{k}r}$ are $\dot{q}_{\mathbf{k}r} = \partial s_{\mathbf{k}}/\partial q_{\mathbf{k}r}$ (with $\psi_{\mathbf{k}} = |\psi_{\mathbf{k}}|\,\mathrm{e}^{\mathrm{i}s_{\mathbf{k}}}$), just as in the pilot-wave theory of a nonrelativistic particle of unit mass in a harmonic oscillator potential in the $q_{\mathbf{k}1} - q_{\mathbf{k}2}$ plane. Thus we deduce that, in the absence of gravity, for a single mode

$\mathbf{k}$ in a superposition of many different states of definite occupation number, the probability distribution $\rho_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)$ will rapidly relax to equilibrium, $\rho_{\mathbf{k}} \to |\psi_{\mathbf{k}}|^2$ (on a coarse-grained level, again given appropriate initial conditions).

Now, in a flat expanding universe, again with metric $d\tau^2 = dt^2 - a^2\, d\mathbf{x}^2$, the pilot-wave equations for a decoupled mode become [19]

$$\mathrm{i}\frac{\partial \psi_{\mathbf{k}}}{\partial t} = -\frac{1}{2a^3}\left(\frac{\partial^2}{\partial q_{\mathbf{k}1}^2} + \frac{\partial^2}{\partial q_{\mathbf{k}2}^2}\right)\psi_{\mathbf{k}} + \frac{1}{2}ak^2\left(q_{\mathbf{k}1}^2 + q_{\mathbf{k}2}^2\right)\psi_{\mathbf{k}} \tag{20}$$

and

$$\dot{q}_{\mathbf{k}1} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}1}}, \qquad \dot{q}_{\mathbf{k}2} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}2}}. \tag{21}$$

How does the presence of $a = a(t)$ affect the time evolution? If $\lambda_{\text{phys}} \ll H^{-1}$, we recover the Minkowski-space evolution—the expansion timescale $H^{-1} \equiv a/\dot{a}$ being much larger than the timescale $\sim\lambda_{\text{phys}}$ (with $c = 1$) over which $\psi_{\mathbf{k}}$ evolves—and so a superposition of many different states of definite occupation number (for the mode $\mathbf{k}$) will again rapidly relax to equilibrium. On the other hand, if $\lambda_{\text{phys}} \gg H^{-1}$, we expect $\psi_{\mathbf{k}}$ and the associated de Broglie-Bohm trajectories to be approximately static over timescales such that $\lambda_{\text{phys}} \propto a(t)$ expands significantly, so that relaxation is suppressed. The spatial expansion then results in a transfer of nonequilibrium to larger lengthscales (as we saw in late-time inflation).

There are then two 'competing' effects: the usual relaxation to equilibrium, and the transfer of nonequilibrium to larger lengthscales. The former dominates for $\lambda_{\text{phys}} \ll H^{-1}$, the latter for $\lambda_{\text{phys}} \gg H^{-1}$. In a radiation-dominated phase, with $a \propto t^{1/2}$, we have $\lambda_{\text{phys}} \propto t^{1/2}$ and $H^{-1} \propto t$. Thus, at sufficiently small times, *all* physical wavelengths are larger than the Hubble radius ($\lambda_{\text{phys}} > H^{-1}$), and the above reasoning suggests that relaxation to equilibrium will be suppressed (until later times when λ_{phys} becomes smaller than H^{-1}). While further study is needed—such as numerical simulations based on (20), (21), and consideration of entangled and also mixed states—we seem to have a mechanism whereby spatial expansion at very early times can suppress the normal relaxation to equilibrium.

Similar conclusions have been arrived at in terms of the pilot-wave theory of particles [9]. If the distribution of particle positions contains nonequilibrium below a certain lengthscale, the spatial expansion will transfer the nonequilibrium to larger lengthscales. Further, a simple estimate $\tau \sim \hbar/kT$ of the relaxation timescale suggests that relaxation will be suppressed when $\tau \gtrsim t_{\text{exp}} \sim (1\ \text{sec})(1\ \text{MeV}/kT)^2$—that is, when $kT \gtrsim 10^{18}$ GeV $\approx 0.1kT_{\text{P}}$ or $t \lesssim 10t_{\text{P}}$ (where T_{P} and t_{P} are respectively the Planck temperature and time). We emphasize that this estimate, while suggestive, is only heuristic.

If relaxation to quantum equilibrium is indeed suppressed at sufficiently early times, in a realistic cosmological model, this raises the exciting possibility that if the universe indeed began in a state of quantum nonequilibrium, then remnants of such nonequilibrium could have survived to the present day—for particles that decoupled at times so early that equilibrium had not yet been reached. Relic gravitons are believed to decouple at $T \sim T_{\text{P}}$, and there may well be other, more exotic particles (associated with physics beyond the standard model) that decoupled soon after T_{P}. A subsequent inflationary era would presumably dilute their density beyond any hope of detection, but in the absence of inflation it is possible that such particles could have a significant abundance today. Further, such relic nonequilibrium particles might annihilate or decay, producing nonequilibrium photons—which could be detected directly, and tested for violations of Malus' law or for anomalous diffraction and interference patterns.

5. Tests of quantum theory with black holes

According to pilot-wave theory, once quantum equilibrium is reached it is not possible to escape from it (leaving aside the remote possibility of rare fluctuations [7]). A universe in quantum equilibrium is then analogous to a universe stuck in a state of global thermal equilibrium or thermodynamic 'heat death'. Further, in quantum equilibrium it is not possible to harness nonlocality for signalling, just as in global thermal equilibrium it is not possible to convert heat into work [5–8].

However, pilot-wave theory has been well developed only for non-gravitational physics. Indeed, despite much effort, standard quantum theory too has yet to be extended to gravity. It is then conceivable that quantum equilibrium as we know it will turn out to be gravitationally unstable: in a future hidden-variables theory incorporating gravitation, there could exist processes that *generate* quantum nonequilibrium.

One such process might be the formation and evaporation of a black hole, which arguably allows a pure quantum state to evolve into a mixed one [24]. It has been suggested that the resulting 'information loss' (the inability in principle to retrodict the initial state from the final one) could be avoided if the outgoing Hawking radiation were in a state of quantum nonequilibrium, enabling it to carry more information than conventional radiation could [20]. A mechanism has been suggested, whereby (putative) nonequilibrium behind the event horizon is transmitted to the exterior region via the entanglement between the ingoing and outgoing radiation modes [20]. It has also been proposed that the decreased 'hidden-variable entropy' S_{hv} (minus the subquantum H-function, suitably generalized to mixed states [20]) of the outgoing nonequilibrium radiation should balance the increase in von Neumann entropy $S_{\text{vonN}} = -\text{Tr}(\hat{\rho} \ln \hat{\rho})$ associated with the pure-to-mixed transition:

$$\Delta(S_{\text{hv}} + S_{\text{vonN}}) = 0. \tag{22}$$

At the time of writing, the proposed conservation rule (22) is only a simple and somewhat arbitrary hypothesis, relating as it does two very different kinds of entropy, S_{hv} and S_{vonN} (though it has been shown [20] that these entropies must be related even in non-gravitational processes, in ways that need to be explored further). If the pure-to-mixed transition does indeed generate nonequilibrium, it might be hoped that (22) will hold at least as an order-of-magnitude estimate.

The above (obviously speculative) idea could be tested, should Hawking radiation from microscopic black holes ever be observed. Primordial black holes of mass $M \sim 10^{15}$ g are expected to be evaporating today, producing (among other particles) gamma-rays peaked at $\sim$100 MeV [25]. Such radiation has been searched for, so far with no definitive detection, and further searches are under way. Should γ-rays from the evaporation of primordial black holes ever be detected, we propose that their polarization probabilities be carefully checked (for example by Compton polarimetry) for deviations from the standard modulation (3). Another possibility, according to theories with large extra dimensions [26], is that microscopic black holes could be produced in collisions at the TeV scale. If so, their decay products could be tested for deviations from (3).

If the entanglement between ingoing and outgoing Hawking radiation modes does indeed provide a channel for nonlocal information flow from behind the horizon, then one would expect a similar process to occur if, for an 'EPR-pair' initially in the exterior region, one half of the entangled state fell behind the horizon. For an ensemble of such pairs, the particles left in the exterior region should evolve away from quantum equilibrium—by an amount that can be estimated from the proposed rule (22) (where ΔS_{vonN} is obtained by tracing over the infalling particles).

It has been argued that, if the information loss envisaged by Hawking is to be avoided by some form of nonlocal information flow, then such flow must occur even while the hole is still macroscopic [27]. Similar arguments lead us to conclude that, even for a *macroscopic* black hole, allowing one half of an EPR-pair to fall behind the horizon will cause the other half to evolve away from quantum equilibrium—over a timescale small compared to the evaporation timescale [20].

This motivates us to propose another test. Most galactic nuclei contain a supermassive black hole ($M \sim 10^6 - 10^{10} M_\odot$) surrounded by a thin accretion disc [28]. It is well established that x-ray emission lines, in particular the Kα iron line at 6.4 keV, may be used to probe the spacetime geometry in the strong gravity region close to the event horizon [29]. The intrinsically narrow line is broadened and skewed by relativistic effects, with an extended red wing caused by the gravitational redshift of photons emitted from very near the horizon. This much is well known. Now, the idea is to identify an atomic *cascade* emission that generates entangled photon pairs at small radii, such that a significant fraction of the photons reaching Earth have partners that fell behind the horizon. Polarization measurements of the received photons would then provide a test of Malus' law (3)—and a probe of possible nonequilibrium, for example in the form of a quadrupole probability law (6)—for photons entangled with partners inside the black hole.

The feasibility of this experiment has been discussed in detail elsewhere [20, 30]. Here, we summarize what appear to be the main points:

- In a $0-1-0$ two-photon cascade, for example, the polarization state shows a strong and phase-coherent entanglement only if the emitted momenta are approximately antiparallel [31]. This may be realized in our experiment by restricting attention to photons with the largest redshift: these have emission radii r_e closest to the horizon at $r_+ = M + \sqrt{M^2 - a^2}$ (where a is the specific angular momentum of the hole), and as $r_e \to r_+$ the photons will escape—and avoid being absorbed by the hole or the accretion disc—only if they are directed parallel to the surface of the disc [32].
- The effect will be *diluted* by received photons with (a) no cascade partners, (b) cascade partners that were not captured by the black hole, (c) cascade partners that were captured but did not have appropriately directed momenta at the point of emission.
- Scattering along the line of sight could degrade the entanglement between the outgoing and ingoing photons, and might cause relaxation $\rho(\lambda) \to \rho_{QT}(\lambda)$. This may be minimized by an appropriate choice of photon frequency and by choosing an accretion disc viewed face-on (with a clear line of sight to the central black hole).
- If the nonequilibrium distribution $\rho(\lambda) \neq \rho_{QT}(\lambda)$ for the received photons depends on the spatial location of the emission, the sought-for effect could be smeared out by spatial averaging over the emitting region. If instead $\rho(\lambda)$ is independent of location, such averaging will have no effect.
- Only about 0.6% of the observed Kα photons are expected to have Lα cascade partners [20, 30]. We hope that other relativistically broadened lines will be discovered, with a larger fraction of cascade partners[3].

Note that true deviations from (3) may be distinguished from ordinary noise and experimental errors by comparing results from the astronomical source with results from a laboratory source. Also, if the effect exists, it will be larger towards the red end of the (broadened) emission line, because these photons are emitted closer to the horizon and are therefore more likely to have partners that were captured.

[3] Broadened lines from oxygen, nitrogen and carbon have in fact already been reported [33].

6. Neutrino oscillations

Microscopic quantum-gravitational effects might induce a pure-to-mixed evolution of the quantum state in a system of oscillating neutrinos, resulting in damping and decoherence effects that might be observable over astrophysical and cosmological (or even just atmospheric) path lengths—see, for example, [33–39]. Such evolution may be modelled by corrections to the usual unitary evolution of the density operator $\hat{\rho}(t)$. Writing

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -\mathrm{i}[\hat{H}, \hat{\rho}] - \mathcal{D}(\hat{\rho}),$$

the extra term $\mathcal{D}$ breaks the usual conservation of $\mathrm{Tr}(\hat{\rho}^2)$. It is usually assumed that $\mathcal{D}$ takes a Lindblad form, and that the mean energy $\mathrm{Tr}(\hat{\rho}\hat{H})$ is conserved. Under the usual assumptions, the term $\mathcal{D}$ generates an increase in von Neumann entropy $S_{\mathrm{vonN}} = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$ over time. (See, for example, [34].)

A detailed phenomenological parameterization of $\mathcal{D}$ has been developed, and extensive comparisons with data have been made [34, 35, 39, 40]. If we follow the hypotheses of section 5 (assuming that $\mathcal{D}$ originates, for example, from the formation and evaporation of microscopic black holes), then any such pure-to-mixed transition will generate quantum nonequilibrium, of a magnitude that might be constrained by (22). This will result in nonequilibrium anomalies in the composition of an oscillating neutrino beam.

Consider the simple case of just two flavours, labelled ν_μ and ν_τ. Lepton number eigenstates $|\nu_\mu\rangle, |\nu_\tau\rangle$ are linear combinations

$$|\nu_\mu\rangle = |\nu_1\rangle\cos\alpha + |\nu_2\rangle\sin\alpha, \qquad |\nu_\tau\rangle = -|\nu_1\rangle\sin\alpha + |\nu_2\rangle\cos\alpha$$

of mass eigenstates (masses m_1, m_2) where α is the mixing angle. For a beam of energy $E \gg m_1, m_2$, terms in $|\nu_1\rangle$ and $|\nu_2\rangle$ propagate with relative phases $\mathrm{e}^{\mathrm{i}kt/2}$ and $\mathrm{e}^{-\mathrm{i}kt/2}$ respectively, where $k \equiv \left(m_2^2 - m_1^2\right)/2E$ [41].

The oscillating two-state system may be represented in Bloch space, with $|\nu_1\rangle$ and $|\nu_2\rangle$ corresponding to unit vectors respectively up and down the z-axis. We then have a Hamiltonian $\hat{H} = -(k/2)\hat{\sigma}_z$ (where $\hat{\sigma}_z$ is a Pauli operator). For an arbitrary density operator $\hat{\rho} = \frac{1}{2}(\hat{I} + \mathbf{P}\cdot\hat{\boldsymbol{\sigma}})$, the mean polarization $\mathbf{P} = \mathrm{Tr}(\hat{\rho}\hat{\boldsymbol{\sigma}})$ then evolves as $\mathrm{d}\mathbf{P}/\mathrm{d}t = \mathbf{k}\times\mathbf{P}$ where $\mathbf{k} \equiv (0, 0, -k)$. An initial pure state $\hat{\rho}(0) = |\nu_\mu\rangle\langle\nu_\mu|$ with

$$\mathbf{P}(0) = (\sin 2\alpha, 0, \cos 2\alpha) \tag{23}$$

evolves into a pure state with

$$\mathbf{P}(t) = (\sin 2\alpha\cos kt, -\sin 2\alpha\sin kt, \cos 2\alpha) \tag{24}$$

(where $|\mathbf{P}(t)| = 1$), and the quantum survival probability for ν_μ shows the well-known oscillation

$$p_{\mathrm{QT}}^{\mu}(t) = \mathrm{Tr}(\hat{\rho}(t)|\nu_\mu\rangle\langle\nu_\mu|) = 1 - \tfrac{1}{2}(1 - \cos kt)\sin^2 2\alpha$$

over a neutrino path length $l \simeq t$.

In the simplest generalization to a pure-to-mixed evolution, we have [34]

$$\dot{P}_x = kP_y - \gamma P_x, \qquad \dot{P}_y = -kP_x - \gamma P_y, \qquad \dot{P}_z = 0,$$

where $\gamma \geqslant 0$ is a phenomenological parameter. An initial pure state $\hat{\rho}(0) = |\nu_\mu\rangle\langle\nu_\mu|$ now evolves into a mixed state with

$$\mathbf{P}(t) = (\mathrm{e}^{-\gamma t}\sin 2\alpha\cos kt, -\mathrm{e}^{-\gamma t}\sin 2\alpha\sin kt, \cos 2\alpha) \tag{25}$$

(where now $|\mathbf{P}(t)| < 1$), and the oscillations in the survival probability

$$p_{\mathrm{QT}}^{\mu}(t) = 1 - \tfrac{1}{2}(1 - \mathrm{e}^{-\gamma t}\cos kt)\sin^2 2\alpha$$

are damped over distances $l \gtrsim 1/\gamma$. The (initially zero) von Neumann entropy $S_{\rm vonN}(t)$ increases with time, reaching a limiting value

$$S_{\rm vonN}(\infty) = -\cos^2\alpha \ln(\cos^2\alpha) - \sin^2\alpha \ln(\sin^2\alpha).$$

If such pure-to-mixed transitions exist, it is possible that they are accompanied by a transition from quantum equilibrium to quantum nonequilibrium, along the lines considered in section 5. Applying the ansatz (22), the nonequilibrium distribution would satisfy the constraint

$$S_{\rm hv}(t) = -S_{\rm vonN}(t), \tag{26}$$

where in a general hidden-variables theory $S_{\rm hv}$ takes the form

$$S_{\rm hv} = -\int {\rm d}\lambda\rho \ln(\rho/\rho_{\rm QT}).$$

According to (26), the hidden-variable entropy $S_{\rm hv}$ decreases with path length $l \simeq t$, in a manner that is fully determined by the dynamics of the pure-to-mixed transition.

Quantum nonequilibrium $\rho(\lambda) \neq \rho_{\rm QT}(\lambda)$ would change the composition of a neutrino beam, in a manner depending on the details of the hidden-variables theory. Generally speaking, the quantum survival probability for ν_μ may be written as

$$p^\mu_{\rm QT}(t) = \tfrac{1}{2}(1 + \mathbf{P}(0)\cdot\mathbf{P}(t)), \tag{27}$$

which is again Malus' law (4) for a two-state system: $p^\mu_{\rm QT}(t)$ is just the probability $p^+_{\rm QT}(\mathbf{m})$ at time t for an 'up' outcome of a quantum measurement along the axis specified by the unit vector $\mathbf{m} = \mathbf{P}(0)$ in Bloch space (corresponding to measuring for the presence of ν_μ), where the measurement is carried out on a system with polarization $\mathbf{P}(t)$. As discussed in section 2, the probability law (4) is equivalent to expectation additivity for incompatible observables, and both are generically violated in nonequilibrium [12].

For example, applying the quadrupole probability law (6) to the case at hand, we have a nonequilibrium survival probability for ν_μ,

$$p^\mu(t) = p^\mu_{\rm QT}(t) + \tfrac{1}{2}(\mathbf{P}(0)\cdot\mathbf{b}(t))(\mathbf{P}(0)\cdot\mathbf{P}(t)), \tag{28}$$

where $\mathbf{P}(0)$ and $\mathbf{P}(t)$ are given by (23) and (25) respectively, and where the time dependence of $\mathbf{b}(t)$ (with $\mathbf{b}(0) = 0$) corresponds to the generation of nonequilibrium during the pure-to-mixed transition (perhaps in accordance with the constraint (26)). In the limit $t \to \infty$, for example, the composition of the beam is shifted from the quantum ν_μ fraction

$$p^\mu_{\rm QT}(\infty) = 1 - \tfrac{1}{2}\sin^2 2\alpha \tag{29}$$

to the nonequilibrium ν_μ fraction

$$p^\mu(\infty) = p^\mu_{\rm QT}(\infty) + \tfrac{1}{2}(b_x(\infty)\sin 2\alpha + b_z(\infty)\cos 2\alpha)\cos^2 2\alpha. \tag{30}$$

7. Particles from very distant sources

Finally, we consider a method for testing quantum probabilities at tiny lengthscales, a method that is based on the huge spreading of the wave packet for particles emitted by very distant (astrophysical or cosmological) sources. In the right circumstances, such spreading can cause microscopic deviations from the Born rule (if they exist) to be expanded up to observable lengthscales. We shall restrict ourselves here to the case of pilot-wave theory, though the argument can be generalized. As we shall see, there are a number of practical difficulties with this method, and it is unclear whether they could all be overcome in a real experiment. Still, the idea might be worth considering further.

To explain the basic mechanism, we first consider a single nonrelativistic particle (labelled i) in free space, with initial wavefunction $\psi_i(\mathbf{x},0)$ (at $t=0$) localized around $\mathbf{x}_i$ with a width $\Delta_i(0)$, where at later times $\psi_i(\mathbf{x},t)$ spreads out to a width $\Delta_i(t)$. For large t, we have approximately $\Delta_i(t) \sim \hbar t/(m\Delta_i(0))$ (where $\sim\hbar/\Delta_i(0)$ is the initial quantum momentum spread). One might think, for example, of a spreading Gaussian packet. Now consider (in pilot-wave theory) the time evolution of an initial distribution $\rho_i(\mathbf{x},0)$ that differs from $|\psi_i(\mathbf{x},0)|^2$ at a 'nonequilibrium lengthscale' $\delta(0)$ at $t=0$. (We mean this in the following sense: if $\rho_i(\mathbf{x},0)$ and $|\psi_i(\mathbf{x},0)|^2$ are each coarse grained or averaged over a volume ε^3, the difference between them is erased if and only if $\varepsilon \gg \delta(0)$.) Because ρ_i and $|\psi_i|^2$ obey the same continuity equation, with the same (de Broglian) velocity field, the ratio $f_i(\mathbf{x},t) \equiv \rho_i(\mathbf{x},t)/|\psi_i(\mathbf{x},t)|^2$ is conserved along particle trajectories (where nonequilibrium corresponds to $f_i \neq 1$). Thus, along a trajectory $\mathbf{x}(t) \equiv g_t(\mathbf{x}(0))$ we have $f_i(\mathbf{x}(t),t) = f_i(\mathbf{x}(0),0)$, and the distribution at time t may be written as

$$\rho_i(\mathbf{x},t) = |\psi_i(\mathbf{x},t)|^2 f_i\left(g_t^{-1}(\mathbf{x}),0\right), \tag{31}$$

where g_t^{-1} is the inverse map from $\mathbf{x}(t)$ to $\mathbf{x}(0)$. If the map $g_t : \mathbf{x}(0) \to \mathbf{x}(t)$ is essentially an expansion—with small (localized) volumes V_0 of $\mathbf{x}(0)$-space being mapped to large volumes V_t of $\mathbf{x}(t)$-space—then the inverse map $g_t^{-1} : \mathbf{x}(t) \to \mathbf{x}(0)$ will be essentially a compression. And because the spreading of $|\psi_i|^2$ is precisely the spreading of an initial equilibrium distribution by the same map g_t, the factor by which g_t expands an initial volume will be approximately $\sim(\Delta_i(t)/\Delta_i(0))^3$, so that $V_t \sim (\Delta_i(t)/\Delta_i(0))^3 V_0$. Thus, if $f_i(\mathbf{x},0)$ deviates from unity on a lengthscale $\delta(0)$, then $f_i(g_t^{-1}(\mathbf{x}),0)$ will deviate from unity on an expanded lengthscale

$$\delta(t) \sim (\Delta_i(t)/\Delta_i(0))\delta(0). \tag{32}$$

Therefore, from (31), the distribution $\rho_i(\mathbf{x},t)$ at time t will show deviations from $|\psi_i(\mathbf{x},t)|^2$ on the *expanded* nonequilibrium lengthscale $\delta(t)$ [8, 9].

As a simple example (assuming that the above nonrelativistic reasoning extends to photons in some appropriate way), consider a photon with an initial wave packet width $\Delta_i(0) \sim 10^{-6}$ cm, emitted by an atom in the neighbourhood of a quasar at a distance $d \sim 10^{27}$ cm. The expansion factor is $\Delta_i(t)/\Delta_i(0) \sim d/\Delta_i(0) \sim 10^{33}$, and an initial nonequilibrium lengthscale of (for example) $\delta(0) \sim 10^{-33}$ cm is expanded up to $\delta(t) \sim 1$ cm. (A photon would of course be found on the surface of a sphere of radius ct, but distances on the spherical surface still expand by a factor $\sim d/\Delta_i(0)$.)

So far we have considered the ideal case of a pure ensemble of identical initial wavefunctions $\psi_i(\mathbf{x},0)$ centred around the same point $\mathbf{x}_i$ and expanding in free space. To be realistic, we need to consider a mixed ensemble emitted by a source of finite spatial extent[4] and propagating in a tenuous (intergalactic) medium.

Let the initial density operator be a mixture

$$\hat{\rho}(0) = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

of wavefunctions $\psi_i(\mathbf{x},0)$ centred at different points $\mathbf{x}_i$, with p_i being the probability for the ith state. For simplicity, let us first assume that $\psi_i(\mathbf{x},0) = \psi(\mathbf{x}-\mathbf{x}_i,0)$, so that we have a mixture with the 'same' wavefunction spreading out from different locations $\mathbf{x}_i$. The quantum equilibrium probability density at time t is

$$\rho_{QT}(\mathbf{x},t) = \langle\mathbf{x}|\hat{\rho}(t)|\mathbf{x}\rangle = \sum_i p_i |\psi(\mathbf{x}-\mathbf{x}_i,t)|^2. \tag{33}$$

[4] Averaging over the spatial extent of the source is important here because the hidden variables are particle positions—and not some more abstract (or perhaps internal) degrees of freedom λ whose distribution might be independent of the spatial location of the emission.

For each (quantum-theoretically) pure subensemble with guiding wavefunction $\psi_i(\mathbf{x}, t)$, we may define an actual distribution $\rho_i(\mathbf{x}, t)$ (generally distinct from $|\psi_i(\mathbf{x}, t)|^2$), while for the whole ensemble the distribution may be written as

$$\rho(\mathbf{x}, t) = \sum_i p_i \rho_i(\mathbf{x}, t)$$

(where in general $\rho(\mathbf{x}, t) \neq \rho_{\mathrm{QT}}(\mathbf{x}, t)$). Let us also assume that, at $t = 0$, each $\rho_i(\mathbf{x}, 0)$ takes the form $\rho_i(\mathbf{x}, 0) = \pi(\mathbf{x} - \mathbf{x}_i, 0)$, where $\pi(\mathbf{x} - \mathbf{x}_i, 0)$ deviates from $|\psi(\mathbf{x} - \mathbf{x}_i, 0)|^2$ at a nonequilibrium lengthscale $\delta(0)$, so that we have a mixture with the 'same' nonequilibrium distribution $\pi(\mathbf{x} - \mathbf{x}_i, t)$ spreading out from different locations $\mathbf{x}_i$. (In work to be published elsewhere, we shall consider dropping this last assumption.) The ensemble distribution is then

$$\rho(\mathbf{x}, t) = \sum_i p_i \pi(\mathbf{x} - \mathbf{x}_i, t). \tag{34}$$

From our discussion of the pure case, we know that $\pi(\mathbf{x}-\mathbf{x}_i, t)$ will deviate from $|\psi(\mathbf{x}-\mathbf{x}_i, t)|^2$ on an expanded lengthscale $\delta(t) \sim (\Delta(t)/\Delta(0))\delta(0)$, where $\Delta(t)$ is the width of $|\psi|^2$ at time t.

Will a similar difference be visible between the spatially averaged distributions (33) and (34)? The answer depends on whether the linear size R of the source is larger or smaller than the (pure) expanded nonequilibrium lengthscale $\delta(t)$. If $R \gg \delta(t)$, the spatial averaging will erase the differences between $\pi(\mathbf{x}-\mathbf{x}_i, t)$ and $|\psi(\mathbf{x}-\mathbf{x}_i, t)|^2$, resulting in $\rho(\mathbf{x}, t) \approx \rho_{\mathrm{QT}}(\mathbf{x}, t)$. If, on the other hand, $R \lesssim \delta(t)$, the spatial averaging cannot erase the nonequilibrium, and the observed ensemble distribution $\rho(\mathbf{x}, t)$ will deviate from the quantum expression $\rho_{\mathrm{QT}}(\mathbf{x}, t)$ on the expanded lengthscale $\delta(t)$.

We then arrive at the following conclusion (tentatively ignoring the effects of scattering and of a mixture of different wavefunctions $\psi_i(\mathbf{x}, 0) \neq \psi(\mathbf{x} - \mathbf{x}_i, 0)$). For a distant source of linear extension R, the spreading of wave packets from an initial width $\Delta(0)$ to a larger width $\Delta(t)$ will generate an observable expansion of the nonequilibrium lengthscale from $\delta(0)$ to $\delta(t) \sim (\Delta(t)/\Delta(0))\delta(0)$, provided the 'no smearing' condition

$$R \lesssim \delta(t) \tag{35}$$

is satisfied.

Before examining the feasibility of (35) ever being satisfied in practice, let us first indicate how our analysis—carried out so far in free space—may be extended to include the effect of scattering by the tenuous intergalactic medium.

Our strategy is as follows. We write the perturbed de Broglie–Bohm trajectory $\mathbf{x}(t)$ (guided by a perturbed wavefunction that includes scattering terms) as $\mathbf{x}(t) = \mathbf{x}_{\mathrm{free}}(t) + \delta\mathbf{x}(t)$, where $\mathbf{x}_{\mathrm{free}}(t)$ denotes the trajectory in free space. As we have seen, the spreading of the trajectories $\mathbf{x}_{\mathrm{free}}(t)$ generates an expanding nonequilibrium lengthscale $\delta(t)$. The question is: will the trajectory perturbations $\delta\mathbf{x}(t)$ cause the expanding nonequilibrium to relax to equilibrium? Considering again the property of de Broglian dynamics, that (for a pure subensemble) $f \equiv \rho/|\psi|^2$ is conserved along trajectories, a little thought shows that a necessary condition for the erasure of nonequilibrium on the expanded lengthscale $\delta(t)$ is that the perturbations $\delta\mathbf{x}(t)$ have a magnitude at least comparable to $\delta(t)$. If, on the contrary,

$$|\delta\mathbf{x}(t)| \ll \delta(t), \tag{36}$$

it will be impossible for the perturbations to erase the expanding nonequilibrium—simply because the trajectories will not be able to distribute the values of f in a manner required for the distributions ρ and $|\psi|^2$ to become indistinguishable on a coarse-graining scale of order $\delta(t)$.

A straightforward argument suggests that the 'no relaxation' condition (36) is indeed likely to be realized in practice. To estimate the magnitude $|\delta\mathbf{x}(t)|$, at large distances from the source we may approximate the wavefunction as a plane wave $e^{i\mathbf{k}\cdot\mathbf{x}}$ incident on a tenuous medium modelled by fixed scattering centres with positions $\mathbf{x}_s$. In a time-independent description of the scattering process, each scattering centre (associated with some potential) contributes a scattered wave which, at large distances from $\mathbf{x}_s$, takes the asymptotic form $f_s(\theta,\phi)\,e^{ikr_s}/r_s$, where $r_s \equiv |\mathbf{x}-\mathbf{x}_s|$ and (θ,ϕ) are standard angular coordinates defined relative to $\mathbf{k}$ as the 'z-axis'. The scattering amplitude $f_s(\theta,\phi)$ is related to the differential cross section by the usual formula $d\sigma_s/d\Omega = |f_s(\theta,\phi)|^2$. For simplicity we may consider identical and isotropic scattering centres: $f_s(\theta,\phi) = f = \text{const.}$ for all s, so that $f^2 = \sigma/4\pi$ where σ is the cross section. The total (time-independent) wavefunction is then

$$\psi(\mathbf{x}) = e^{i\mathbf{k}\cdot\mathbf{x}} - \frac{1}{2}\sqrt{\frac{\sigma}{\pi}}\sum_s e^{i\mathbf{k}\cdot\mathbf{x}_s}\frac{e^{ikr_s}}{r_s} \tag{37}$$

(where $e^{i\mathbf{k}\cdot\mathbf{x}_s}$ is a relative phase for each source). We assume that the scattering centres are more or less uniformly distributed in space with a number density n and mean spacing $(1/n)^{1/3}$. The intergalactic medium is mainly composed of ionized hydrogen, with an electron number density $n \sim 10^{-7}$ cm^{-3} and mean spacing $\sim$200 cm. For most cases of interest, the incident wavelength $2\pi/k$ will be much smaller than 200 cm, justifying use of the asymptotic form $\sim e^{ikr_s}/r_s$ for the scattered waves. (In an appropriate extension of this nonrelativistic model to photons, wavelengths $2\pi/k \gtrsim 200$ cm correspond to radio waves.) In this approximation, the de Broglie–Bohm trajectories take the form

$$\mathbf{x}(t) = \mathbf{x}(0) + (\hbar\mathbf{k}/m)t + \delta\mathbf{x}(t),$$

where $\delta\mathbf{x}(t)$ is a small perturbation. We expect $\delta\mathbf{x}(t)$ to behave like a random walk, with $|\delta\mathbf{x}(t)| \propto \sqrt{t}$. If this is the case, then because $\delta(t) \propto \Delta(t) \propto t$ (for large t), the no relaxation condition (36) will necessarily be satisfied for sufficiently large t. It then appears that scattering by the intergalactic medium is unlikely to offset the expansion of the nonequilibrium lengthscale.

In contrast, the no smearing condition (35) is very severe, and is unlikely to be realized except in special circumstances. In our example above, of a photon emitted by an atom in the vicinity of a quasar, the expansion factor $\Delta(t)/\Delta(0) \sim 10^{33}$ suggests that the tantalizing Planck lengthscale $l_P \sim 10^{-33}$ cm at the time of emission may be within reach of experiments performed on the detected photon now at a lengthscale $\sim$1 cm. Unfortunately, according to (35) any nonequilibrium at the Planck scale would be smeared out unless the source had a size $R \lesssim 1$ cm, which seems much too small to be resolvable in practice (at the assumed distance d $\sim 10^{27}$ cm).

This seemingly insurmountable obstacle could perhaps be overcome, however, by considering a combination of: (a) shorter wavelengths, corresponding to a smaller $\Delta(0)$ and a larger $\delta(t)$; and (b) special astrophysical circumstances in which remarkably small sources can in fact be resolved.

As an example of (a), one may consider a gamma-ray emission (say from an atomic nucleus) with $\Delta(0) \sim 10^{-12}$ cm, again from a distance $d \sim 10^{27}$ cm, yielding an expansion factor $\Delta(t)/\Delta(0) \sim d/\Delta(0) \sim 10^{39}$. To probe the Planck scale ($\delta(0) \sim 10^{-33}$ cm) then requires a source size $R \lesssim \delta(t) \sim 10^6$ cm = 10 km, which is comparable to what is believed to be the size of the central engine of a typical gamma-ray burst [42]. (Photons from the central engine of a gamma-ray burst are not normally expected to propagate essentially freely immediately after emission, and there are in any case many uncertainties concerning the mechanism of such bursts; even so, the example just quoted does suggest that the no smearing constraint (35) might in fact be satisfied by a judicious choice of wavelength and source.)

As examples of (b), we quote the following instances of remarkably small sources that either have already been resolved in practice, or that might be in the near future:

- Nanosecond radio bursts have been observed coming from the Crab pulsar [43]. The observations have a time resolution $\Delta t \approx 2$ ns, corresponding to an emitting source diameter $\lesssim c\Delta t \approx 60$ cm. Isolated sub-pulses were detected at this time resolution, and interpreted as caused by the collapse of highly localized ($\sim$60 cm) structures in a turbulent plasma. Whatever their nature, these objects are the smallest ever resolved outside the solar system. For our purposes, the Crab pulsar is too close ($d \sim 10^{22}$ cm) and radio wavelengths are too large. Even so, it is clear that the detection of transients on very small timescales—at an appropriate distance and wavelength—offers a way of resolving sources satisfying the no smearing condition (35).
- An ultraviolet ($\approx$170 eV) 'hotspot' of radius $\lesssim$60 m has been detected on the surface of the Geminga pulsar (at a distance $d \sim 10^{21}$ cm from Earth) [44]. While the source is again too close for our purposes, both the wavelength and the source size are promising.
- Microsecond gamma-ray bursts of energy $\gtrsim$100 MeV (or wavelength $\lesssim 10^{-12}$ cm)—which might originate from exploding primordial black holes—should be observable with the SGARFACE experiment [45]. A burst time structure with resolution $\Delta t \approx 10^{-6}$ s corresponds to a source size $\lesssim c\Delta t \approx 10^4$ cm $= 0.1$ km. Microsecond gamma-ray bursts at cosmological distances (d $\sim 10^{27} - 10^{28}$ cm) would then seem to satisfy our criteria (except that the possibility of essentially free propagation from emission to detection still needs to be considered as well).

Finally, we must consider dropping the simplifying assumption that the packets $\psi_i(\mathbf{x}, 0)$ emitted by the source differ only in their initial location. In general, we will have $\psi_i(\mathbf{x}, 0) \neq \psi(\mathbf{x} - \mathbf{x}_i, 0)$, with different packets ψ_i emitted from different locations $\mathbf{x}_i$. Here we seem to encounter the most severe practical problem of all. As in the case of perturbations from the intergalactic medium, a necessary condition for the different wavefunctions to lead to an erasure of nonequilibrium on the expanded lengthscale $\delta(t)$ is that trajectories $\mathbf{x}_i(t)$, $\mathbf{x}_j(t)$ (with the same initial point $\mathbf{x}(0)$) generated by respective wavefunctions $\psi_i(\mathbf{x}, t)$, $\psi_j(\mathbf{x}, t)$ should differ by an amount at least comparable to $\delta(t)$. If instead

$$|\mathbf{x}_i(t) - \mathbf{x}_j(t)| \lesssim \delta(t) \tag{38}$$

for all i, j (and for all $\mathbf{x}(0)$), it will be impossible for the expanding nonequilibrium to be erased upon averaging over the mixture of wavefunctions.

Unfortunately, it is unclear whether the 'no mixing' condition (38) could ever be realized in practice. The wavefunctions $\psi_i(\mathbf{x}, t)$, $\psi_j(\mathbf{x}, t)$ would have to be almost the same, to an extremely high accuracy, in order to generate trajectories $\mathbf{x}_i(t)$, $\mathbf{x}_j(t)$ satisfying (38). To see this, as a rough estimate one may take $\mathbf{x}_i(t) \sim (\Delta\mathbf{p}_i/m)t$, where $\Delta\mathbf{p}_i$ is the quantum momentum spread for the wave function ψ_i; and similarly for $\mathbf{x}_j(t)$. Condition (38) then reads

$$|\Delta\mathbf{p}_i - \Delta\mathbf{p}_j| \lesssim \frac{\hbar}{\Delta(0)}\frac{\delta(0)}{\Delta(0)} \tag{39}$$

(taking all initial packets to have approximately the same width $\Delta(0)$ and nonequilibrium lengthscale $\delta(0)$). Since the factor $\delta(0)/\Delta(0)$ is very tiny, the momentum spreads of the emitted packets must be very tightly constrained, and there seems to be no obvious way in which this could happen.

If the method proposed in this section is to work in practice, some extra ingredient is needed to ensure that (39) is satisfied. At the time of writing, we are unable to say if such an ingredient is likely to be found.

8. Conclusion

We have discussed several proposals for astrophysical and cosmological tests of quantum theory. Our general aim has been to test the foundations of quantum theory in new and extreme conditions, guided in particular by the view that quantum theory is an emergent description of an equilibrium state. While we have often used the pilot-wave theory of de Broglie and Bohm, much of our reasoning applies to general deterministic hidden-variables theories.

Pilot-wave theory is the only hidden-variables theory of broad scope that we possess. Possibly, it is a good approximation to the correct theory; or perhaps it is merely a helpful stepping stone towards the correct theory. Certainly, pilot-wave theory is a simple and natural deterministic interpretation of quantum physics. On the other hand, it could be that the true deterministic hidden-variables theory (if there is one) is quite different, and that in some key respects pilot-wave theory is actually misleading. After all, the observable statistics of the quantum equilibrium state obscure many of the details of the underlying (nonlocal and deterministic) physics. Since all of our experience so far has been confined to the equilibrium state, it would not be surprising if we were led astray in our attempts to construct a subquantum (or hidden-variables) theory. Obviously, many possible theories could underlie the equilibrium physics that we see. The ultimate aim of the proposals made in this paper is to find an empirical window that could help us determine what the true underlying theory actually is.

It is usually assumed that quantum theory is a fundamental framework in terms of which all physical theories are to be expressed. There is, however, no reason to believe *a priori* that quantum theory has an unlimited domain of validity. For 200 years it was generally believed that Newtonian mechanics was a fundamental framework for the whole of physics. Yet, today we know that Newtonian mechanics is merely an emergent approximation (arising from the classical and low-energy limits of quantum field theory). Whether or not quantum theory will suffer a similar fate remains to be seen.

References

[1] Pearle P 1976 *Phys. Rev.* D **13** 857
[2] Pearle P 1979 *Int. J. Theor. Phys.* **18** 489
[3] Pearle P 1989 *Phys. Rev.* A **39** 2277
[4] Ghirardi G-C, Rimini A and Weber T 1986 *Phys. Rev.* D **34** 470
[5] Valentini A 1991 *Phys. Lett.* A **156** 5
[6] Valentini A 1991 *Phys. Lett.* A **158** 1
[7] Valentini A 1992 *PhD Thesis* International School for Advanced Studies, Trieste, Italy [www.sissa.it/ap/PhD/Theses/valentini.pdf]
[8] Valentini A 1996 *Bohmian Mechanics and Quantum Theory: An Appraisal* ed J T Cushing, A Fine and S Goldstein (Dordrecht: Kluwer)
[9] Valentini A 2001 *Chance in Physics: Foundations and Perspectives* ed J Bricmont *et al* (Berlin: Springer)
[10] Valentini A 2002 *Phys. Lett.* A **297** 273
[11] Valentini A 2002 *Pramana J. Phys.* **59** 269
[12] Valentini A 2004 *Phys. Lett.* A **332** 187
[13] Pearle P and Valentini A 2006 *Encyclopaedia of Mathematical Physics* ed J-P Françoise, G Naber and T S Tsun (New York: Elsevier)
[14] de Broglie L 1928 *Électrons et Photons: Rapports et Discussions du Cinquième Conseil de Physique* ed J Bordet (Paris: Gauthier-Villars) (Engl. transl. in [16])
[15] Bohm D 1952 *Phys. Rev.* **85** 166, 180
[16] Bacciagaluppi G and Valentini A 2007 *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference* (Cambridge: Cambridge University Press) (*Preprint* quant-ph/0609184)
[17] Valentini A and Westman H 2005 *Proc. R. Soc.* A **461** 253
[18] Fernström P, Johansson J and Skyman A Chalmers University of Technology unpublished report

[19] Valentini A Inflationary cosmology as a probe of primordial quantum mechanics, in preparation
[20] Valentini A 2004 *Preprint* hep-th/0407032
[21] Liddle A R and Lyth D H 2000 *Cosmological Inflation and Large-Scale Structure* (Cambridge: Cambridge University Press)
[22] Hinshaw G *et al* 2006 *Preprint* astro-ph/0603451
[23] Perez A, Sahlmann H and Sudarsky D 2006 *Class. Quantum Grav.* **23** 2317
[24] Hawking S W 1976 *Phys. Rev.* D **14** 2460
[25] Page D N 1976 *Phys. Rev.* D **13** 198
Page D N 1976 *Phys. Rev.* D **14** 3260
Page D N 1977 *Phys. Rev.* D **16** 2402
[26] Arkani-Hamed N, Dimopoulos S and Dvali G 1998 *Phys. Lett.* B **429** 263
[27] Danielsson U H and Schiffer M 1993 *Phys. Rev.* D **48** 4779
[28] Ferrarese L and Ford H 2005 *Space Sci. Rev.* **116** 523
[29] Reynolds C S and Nowak M A 2003 *Phys. Rep.* **377** 389
[30] Valentini A 2004 *Preprint* astro-ph/0412503
[31] Aspect A 2002 *Quantum (Un)speakables: From Bell to Quantum Information* ed R A Bertlmann and A Zeilinger (Berlin: Springer)
[32] Cunningham C T 1975 *Astrophys. J.* **202** 788
[33] Ogle P M *et al* 2004 *Astrophys. J.* **606** 151
[34] Lisi E, Marrone A and Montanino D 2000 *Phys. Rev. Lett.* **85** 1166
[35] Mavromatos N E 2004 *Preprint* gr-qc/0407005
[36] Anchordoqui L A *et al* 2005 *Phys. Rev.* D **72** 065019
[37] Hooper D, Morgan D and Winstanley E 2005 *Phys. Lett.* B **609** 206
[38] Christian J 2005 *Phys. Rev. Lett.* **95** 160403
[39] Morgan D, Winstanley E, Brunner J and Thompson L F 2006 *Astropart. Phys.* **25** 311
[40] Barenboim G, Mavromatos N E, Sarkar S and Waldron-Lauda A 2006 *Preprint* hep-ph/0603028
[41] Beuthe M 2003 *Phys. Rep.* **375** 105
[42] Mészáros P 2006 *Rep. Prog. Phys.* **69** 2259
[43] Hankins T H, Kern J S, Weatherall J C and Eilek J A 2003 *Nature* **422** 141
[44] Caraveo P A *et al* 2004 *Science* **305** 376
[45] LeBohec S, Krennrich F and Sleege G 2005 *Astropart. Phys.* **23** 235